MTH 201
Multivariable calculus and differential equations
Homework 6
Double integral

1. Evaluate each of the following double integrals over the given rectangular region
   (a) $\iint_D (2x + 4y^3)\, dA,\ D = [0,4] \times [0,3]$
   (b) $\iint_D \frac{1}{2x+3y}\, dA,\ D = [0,1] \times [1,2]$ (HW)
   (c) $\iint_D \frac{1}{1+x+y}\, dA,\ D = [1,3] \times [1,2]$
   (d) $\iint_D x \cos^2 y\, dA,\ D = [0,3] \times [0,\pi/2]$
   (e) $\iint_D x \sin xy\, dA,\ D = [0,1] \times [0,\pi/2]$
   (f) $\iint_D xe^{-xy}\, dA,\ D = [0,2] \times [0,3]$.

2. Find the volume of the solid that lies above the square $Q = [0,2] \times [0,2]$ in $xy-$plane and below the paraboloid $z = 16 - x^2 - y^2$.

3. Find the volume of the solid that lies under the elliptic paraboloid $\frac{x^2}{4} + \frac{y^2}{9} + z = 1$ and above the rectangle $R = [-1,1] \times [-2,2]$ in $xy-$plane.

4. Find the volume of the solid enclosed by the surface $z = 1 + e^x \sin y$ and the planes $x = 1,\ x = -1, y = 0, y = \pi$, and $z = 0$. (HW)

5. Evaluate each of the following double integrals over the given region D
   (a) $\iint_D (6x^2 - 40y)\, dA$, where D is the triangle with vertices $(0,3), (1,1)$, and $(5,2)$
   (b) $\iint_D e^{\frac{x}{y}}\, dA,\ D = \{(x,y) :\ 1 \le y \le 2,\ y \le x \le y^3\}$
   (c) $\iint_D (4xy - y^3)\, dA$, where D is the region bounded by $y = \sqrt{x}$ and $y = x^3$.

6. Find the volume of the solid that lies below the surface given by $z = 16xy + 200$ and lies above the region in the $xy-$plane bounded by $y = x^2$ and $y = 8 - x^2$. (HW)

7. Find the volume of the solid enclosed by the planes $4x + 2y + z = 10, y = 3x, z = 0$, and $x = 0$.

8. Evaluate the following double integrals (use polar co-ordinates)
   (a) $\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{0} \frac{4\sqrt{x^2+y^2}}{1+x^2+y^2}\, dxdy$.
   (b) $\int_0^1 \int_0^{\sqrt{1-y^2}} \cos(x^2 + y^2)\, dxdy$.
   (c) $\iint_D 2xy\, dA$, where D is the portion of the region between the circles of radius 2 and radius 4 centered at the origin that lies in the first quadrant. (HW)
   (d) $\iint_D e^{x^2+y^2}\, dA$, where D is the unit circle centered at the origin.
   (e) $\iint_D (x^2 + y^2)\, dA$, where D is the portion of the unit circle centered at the origin in the first quadrant.
   (f) $\iint_D (3x + 4y^2)\, dA$, where D is the region in the upper half plane bounded by circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$.

9. Find the volume of the solid bounded by the plane $z = 0$ and the parallelopiped $z = 1 - x^2 - y^2$.

10. Find the volume of the region that lies inside $z = x^2 + y^2$ and below the plane $z = 16$.

11. Determine the volume of the region that lies under the sphere $x^2 + y^2 + z^2 = 9$, above the plane $z = 0$, and inside the cylinder $x^2 + y^2 = 5$.

12. Find the area of the region
    (a) that is enclosed by the cardioid $r = 1 + \cos\theta$
    (b) that lies inside the cardioid $r = 1 + \cos\theta$ and outside the circle $r = 1$.
    (c) enclosed by one leaf of the rose $r = 12\cos(3\theta)$.
    (d) cut from the first quadrant by the cardioid $r = 1 + \sin\theta$.